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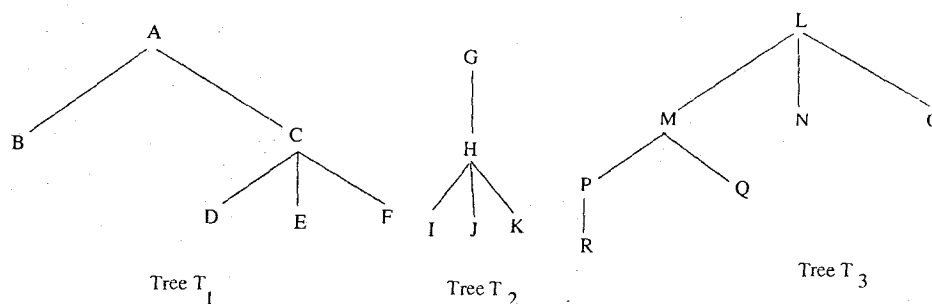
Indian Institute of Technology, Kharagpur

Date..... FN/AN Time: 2 Hrs Full Marks: 30 No. of Students: 65
 Mid (Autumn) Sem 2011-12, Deptt: MA/EC/AE/NA/HS/EX/CH Sub. No. MA 21007
 Subject Name: Design and Analysis of Algorithms

Instruction: Answer all questions.

Question 1 [$1 + 2 + 2 + 1\frac{1}{2} + 1\frac{1}{2} = 8$ marks]

Let F be a forest consisting of trees T_1 , T_2 and T_3 as shown in the following figure.



- Draw the binary tree T corresponding to F .
- List the nodes of F in (i) preorder, (ii) postorder, (iii) inorder, and (iv) level order.
- List the nodes of T in (i) preorder, (ii) postorder, (iii) inorder, and (iv) level order.
- For T_3 , draw its (i) list representation, (ii) left child-right sibling representation, and (iii) degree-two tree representation.
- Draw the internal memory representation of T using (i) sequential representation, (ii) linked representation, and (iii) threaded binary tree representation.

Question 2 [$5+3=8$ marks]

- Write a pseudo-code that returns a pointer to the node (record) containing the k -th largest element in a binary search tree of n elements.
 - Illustrate your algorithm on the following sequence by finding the 3-rd largest element: 33, 37, 24, 16, 23, 20, 11, 15, 17, 22
 - What is the average computing time of your algorithm to find the k -th largest element?
- Show that the number of comparisons required to build a binary search tree from a set of n distinct data items is given by (under the assumption that all possible orderings of n data items are equally likely):

$$T(0) = T(1) = 0$$

$$T(n) = (n-1) + \frac{1}{n} \sum_{i=1}^n (T(i-1) + T(n-i)).$$

- Hence show that $T(n) \leq O(n \log(n))$.

Question 3 [1+3+3+1=8 marks]

- a) Is an array that is in reverse sorted order a heap? Justify your answer.
- b) Write a pseudo-code for creating a heap in linear time. Explain why your algorithm takes linear time.
- c) Illustrate Heapsort on the following sequence: 12, 2, 16, 30, 8, 28, 4, 10, 20, 6, 18
- d) Show that $\sum_{i=1}^k 2^{i-1}(k-i) = 2^k - k - 1, k \geq 1$.

Question 4 [1+1+2+2=6 marks]

- a) Prove that $n\sqrt{n}$ is $O(n^2)$ but not $\Omega(n^2)$ by applying directly the definition of O, Ω .
- b) Prove that a complete binary tree of height k has $2^k - 1$ non-leaf vertices and 2^k leaves.
- c) Prove that the number of different ways to compute the product of $n + 1$ matrices is given by:

$$\binom{1/2}{n+1} (-1)^n 2^{2n+1}$$

- d) Find the optimal matrix ordering that produces the minimum number of integer multiplications to compute the matrix product $ABCDEF$ with the following dimensions:

Matrix	Dimension
A	2×4
B	4×8
C	8×16
D	16×32
E	32×2
F	2×256

-----The End-----